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Virtually-Deterministic Quantum Computing of Nondeterministic Polynomial Problems

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Nondeterministic solutions to complex problems routinely require many fewer resources than deterministic ones. In Turing machines and their equivalents, the deterministic solution requires high energy expenditure because each operation costs energy. We show that photons behave nondeterministically and thus make dramatic energy savings in problems which can be implemented optically.

1. INTRODUCTION

It is common to measure the computational complexity of an algorithm or process in terms of how the computational resources (time, space, energy) must scale with some linear measure N of problem size. In optical processing, N might be the number of input beam resolution cells, output detectors, interconnections, etc. Concentrating on the resource-dominant term, we find that many calculations scale as N^p, where p is some small (often integer) number. We call these polynomial problems or algorithms. Other problems scale in a nonpolynomial way, e.g., p^N, and we call these exponential problems. It is a peculiar feature of exponential problems that they can often be solved by decision-tree algorithms. In such cases, if we magically knew what paths to take (a nondeterministic situation), then we could solve the problem with polynomial resources. Such problems are called nondeterministic polynomial (NP). NP-complete (NPC) problems, a subset of NP, are particularly interesting because each of these problems can be transformed into any other NPC problem with polynomial resources. Thus, if we could ever find a polynomial solution to any one of the several thousand NPC problems, we could solve any other one also with polynomial resources.

Our concern in this paper is with what may be viewed as nondeterministic energy propagation in certain computers. While we wish to exclude lucky

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nondeterministic choices as highly unreliable, we also wish to avoid the difficulty of massive energy costs in making many deterministic calculations. In Section 2, we discuss a heretofore unexplored intermediate case.

In optical computers, the energy required to read out information is manifested as photon detections. We will show that photons behave in precisely the nondeterministic manner described above. Hopfield and Tank (1985) demonstrated that an NPC problem can be implemented as a neural network. Therefore, if we can set up any problem properly for optical processing, the photonic complexity is at most $O(N^2)$, utilizing two spatial dimensions for output.

So far as we know, this represents the first actual implementation of a nondeterministic computation. It also ties together, for the first time, quantum indeterminacy and nondeterministic computation.

2. VIRTUAL PATH CALCULATIONS

There are many paradigms for "explaining" quantum mechanical behavior, all of which are consistent with the theory and the observations. We will move freely and with no consistency among them in our effort to explain the concepts involved. We are thereby not advocating one picture over another.

One popular view is that the wavefunction guides the physical events. In a suitably constructed optical computer (Caulfield and Shamir, 1989, 1991), the configuration of the entire apparatus is required to generate the wavefunction Ψ which predicts, through $\Psi^*\Psi$, the probability of a photon detection at any detector during some time interval. Photons are detected at a point with high probability only if $\Psi^*\Psi$ is high at that point. But, if the apparatus is so constructed that detections represent solutions to a computationally complex problem, then photons behave in the proper nondeterministic manner. They are guided to do so by a wavefunction which has explored all of the possible paths and selected the optimum one.

The viewpoint that all paths are explored and path amplitudes are summed coherently to obtain the wavefunction is the basis of quantum electrodynamics (QED). Indeed, Feynman's exposition of QED (Feynman, 1988) can be read as an explanation of how nature does other nondeterministic computations. An example is light obeying Fermat's principle.

Previous discussions (Caulfield and Shamir, 1989, 1991) of the quantum mechanics of optical computers invoked complementarity, or the wave–particle duality (WPD) principle. The authors even called such computers "WPD processors." In describing the diffraction of a light wave by a spatial light modulator, coded-array aperture, hologram, diffraction grating, pinhole, or whatever, it is usually disadvantageous to invoke the photon picture until a photodetection event has occurred. Then one may speak of the detection of a particle (photon). The region between the source and detector is best regarded as being permeated by the light field, the intensity of which determines the probability of a photon detection at a given spacetime point. For low light levels, the field intensity is low, so the probability of detecting a photon at any given spacetime point is low.

This situation in Young's experiment is frequently the paradigm employed to demonstrate the principle of complementarity (wave-particle duality) and quantum indeterminacy. In a familiar experiment, the light field intensity is so low that no more than one photon is traversing the apparatus at any given time. After waiting a sufficiently long time, we observe a (time-integrated) two-slit interference pattern produced on the detector plane. That implies that, while any given photon could only have traversed the apparatus by passing through one and only one of the two slits, each photon seems to be "aware" of the presence of the other slit. Otherwise, a single-slit diffraction pattern would result. Of course, this dilemma dissolves when the wave picture is invoked for describing the region between source and detector (but not including the detectors, where the photon picture is appropriate). One may think of a field being established between source and detector, disturbed by the presence of the slits in such a way as to provide pathways through spacetime for the photons. Naturally, these paths are influenced by both (or all) slits.

We can quantify these notions and express them mathematically. Consider the path of a photon from one spacetime point $x=(t, \mathbf{x})$ to another $x'=(t', \mathbf{x}')$. The probability amplitude for a particle leaving the point $\mathbf{x}$ at time t and reaching the point $\mathbf{x}'$ at time $t'>t$ is represented by $\langle \mathbf{x}', t' | \mathbf{x}, t\rangle$. Let $\tau \in (t, t')$ but be otherwise arbitrary. Then, we may write

$$\langle \mathbf{x}', t' | \mathbf{x}, t\rangle = \int_{\mathbf{R}^3} \langle \mathbf{x}', t' | \mathbf{x}'', \tau\rangle \langle \mathbf{x}'', \tau | \mathbf{x}, t\rangle \, d^3\mathbf{x}''$$

for $t<\tau<t'$.

We may give a physical interpretation of this expression within the context of the multiple-slit experiment. Suppose a photon is emitted at time t from a point source at the position $\mathbf{x}$. At a later time τ it passes through one of the several apertures in an infinite planar mask $\mathbf{M}$. It ultimately reaches a point $\mathbf{x}'$ in another (detection) plane, parallel to $\mathbf{M}$, at time t'. Then, the probability amplitude for this process is (Manoukian, 1989), to within a normalization factor,

$$\langle \mathbf{x}', t' | \mathbf{x}, t\rangle = \int_{\mathbf{M}} \langle \mathbf{x}', t' | \mathbf{x}'', \tau\rangle \langle \mathbf{x}'', \tau | \mathbf{x}, t\rangle \, d^2\mathbf{x}''$$

where $\mathbf{x}''$ lies in the plane of $\mathbf{M}$ and $t<\tau<t'$. If the mask contains, say, N

apertures $\mathbf{A}_j$, for $j = 1, 2, \ldots, N$, and assuming that a photon cannot arrive at the detection plane unless it passes through one of them, then

$$\langle \mathbf{x}', t' | \mathbf{x}, t \rangle = \sum_{j=1}^{N} \int_{\mathbf{A}_j} \langle \mathbf{x}', t' | \mathbf{x}_j, \tau \rangle \langle \mathbf{x}_j, \tau | \mathbf{x}, t \rangle \, d^2\mathbf{x}_j$$

where $\mathbf{x}_j \in \mathbf{A}_j$.

Thus, the probability amplitude for a photon being emitted from x and arriving at x' is the sum of the amplitudes for all possible paths from x to x' via all the apertures $\mathbf{A}_j$ in the mask **M**. Moreover, the contributions from each of the paths in the sum are simultaneous.

The physical interpretation is that each and every possible path from x to x' through **M** is traversed simultaneously in parallel by many "virtual photons" and the "correct" path corresponds to that of the "real" (detected) photon. These virtual photons make the path explorations with no measurable energy expenditure. Indeed, they are unobservable and exist only for times consistent with the uncertainty principle. It is this "quantum parallelism" (Deutsch, 1985) which endows our quantum computer with its extra-Turing capabilities. Problems having computational complexity beyond the realm of the Turing machine can thus be addressed.

Virtual photons do the computation fully deterministically and in parallel. The observable photons display the results of those calculations probabilistically. A Turing-equivalent computer acts in a deterministic fashion making many serial calculations. Each calculation requires at least an amount $kT \ln 2$ of energy (Brillouin, 1962; Fredkin and Toffoli, 1982). Thus, for a complex problem, energy expenditure can be quite high. However, the energy carriers in optics efficiently appear, in what can be considered to be a nondeterministic fashion, at the proper detectors.

3. CONCLUDING REMARKS

We have explained how to reduce the energy scaling from $O(N^4)$ for typical optics (Caulfield and Shamir, 1989, 1990) to at most $O(N^2)$ and the time complexity (Shamir, 1987; Lohmann and Marathay, 1989) to at most $O(N)$ with optical computers. In the context of Young's experiment, the virtual photons simultaneously search all possible paths through the aperture mask and the selection of the correct solution is manifested as the detection of a "real" photon. The virtual photons employ the deterministic algorithm (searching all paths), while the real photons solve the nondeterministic problem (following the correct path). This can, in principle, be accomplished with as little as a single photon and in as little time as a single time-of-flight, at an energy complexity and time complexity of $O(1)$. The desired precision dictates the actual number of photons required, which is considerably less than for the full deterministic computation.

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